Compilation (#6a) : SSA

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Master 1, ENS de Lyon et Dpt Info, Lyon1

2024-2025

Lyon 1

ENS DE LYON

Credits

Source http://homepages.dcc.ufmg.br/~fernando/classes/dcc888/ementa/slides/StaticSingleAssignment.pdf

- The SSA book (collective)
- Modern Compiler Implementation in C/Java/ML (Andrew Appel)
- Fernando Magno Quintao Pereira's course
  https://www.youtube.com/user/pronesto/videos
- Adrian Sampson's course
  https://www.cs.cornell.edu/courses/cs6120/2020fa/

Motivation: It’s all about information

Compilers alternate between two tasks:

1. *computing* some information (invariants) of the program
2. *using* this information to justify some program transformations

Dataflow analyses associate facts to every program point:
* a fact is *associated to a definition-site* of a variable
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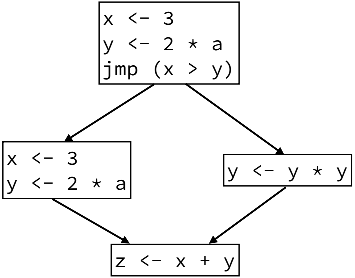

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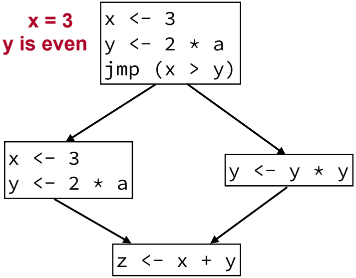

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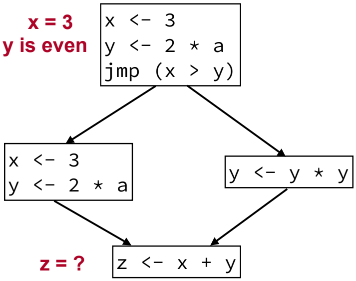

What do we know about x and y?

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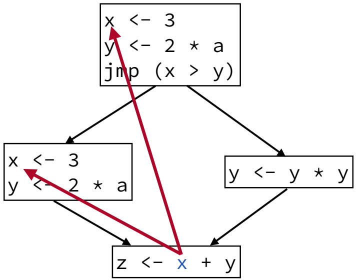
M def and N use of a variable: O(N * M) space and time



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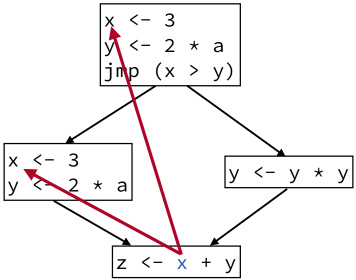

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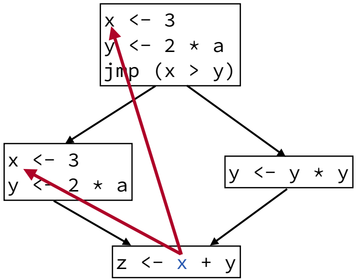

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We want to enforce an invariant by construction: we want an *intermediate representation*

Single Static Assignment (SSA)

Each variable has exactly one definition in the syntax[1]

Use-def chains are explicit in the syntax of the program -> Many optimizations are simplified

[1] Dynamically, it can be defined many times: it is not “Single Assignment”!

Single Static Assignment (SSA)

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Introduced in 1988:
"Global value numbers and redundant computations" by Rosen, Wegman and Zadeck

Used in most modern compilers: GCC, llvm, HotSpot…

We will consider here more specifically Control Flow Graphs in SSA form

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Converting to SSA form: informally

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Converting straight code

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bl:
  a  <- x  + 1
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  a  <- a  + b
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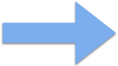

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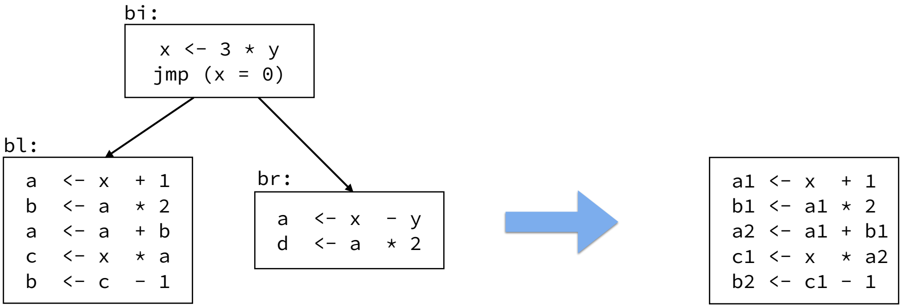
Rule 1: use a fresh index at each def-site

Rule 2: use the most recent definition at each use-site

Converting disjunction points



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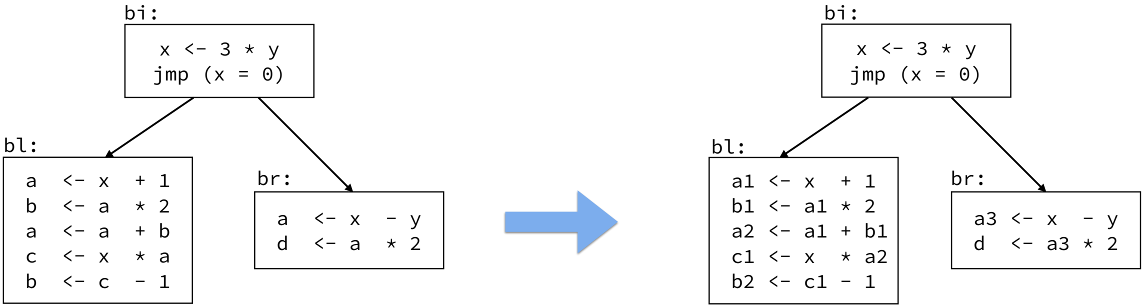

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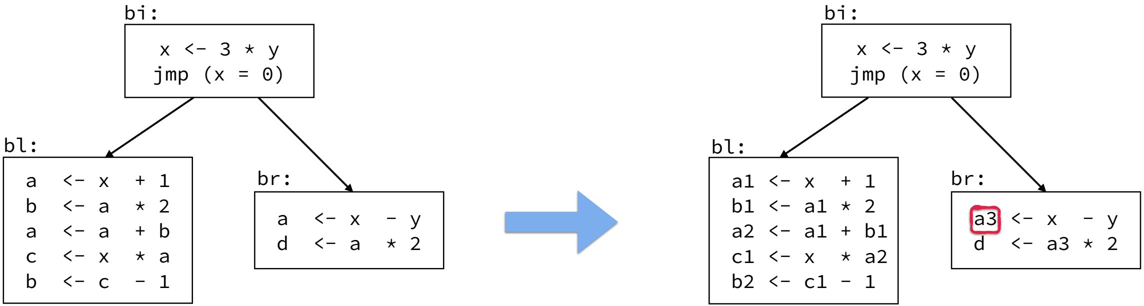

Freshness is global

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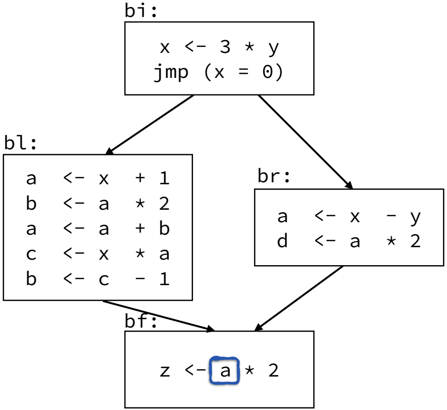
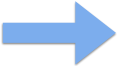
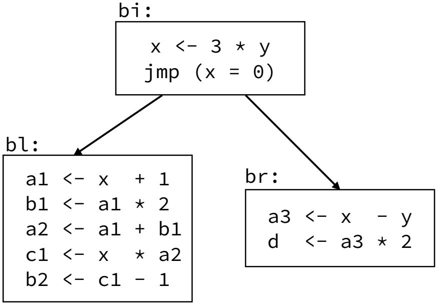
Converting merging points: Φ-nodes







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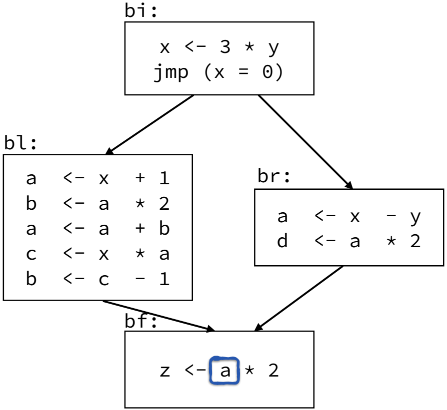

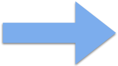

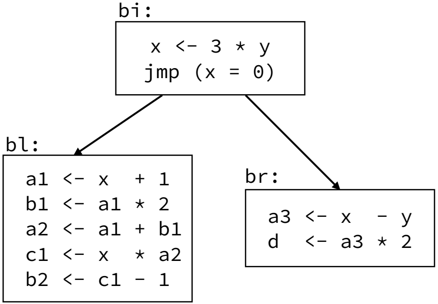

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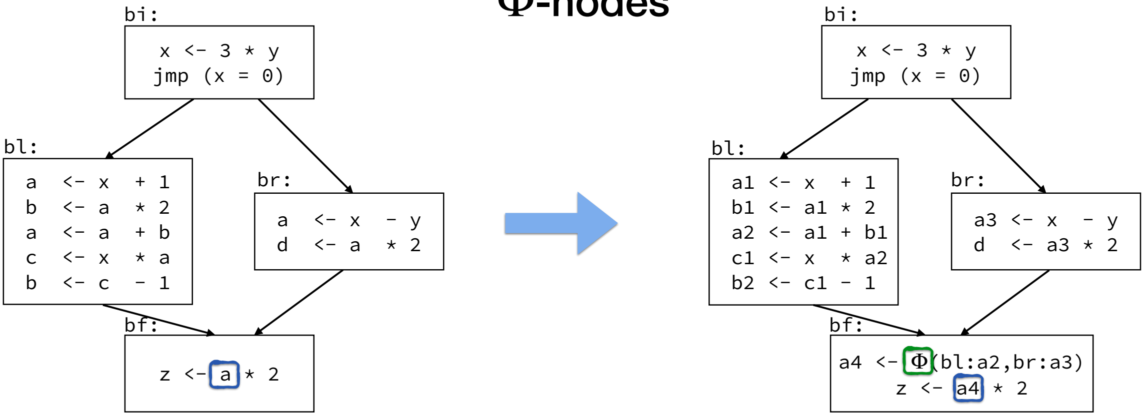

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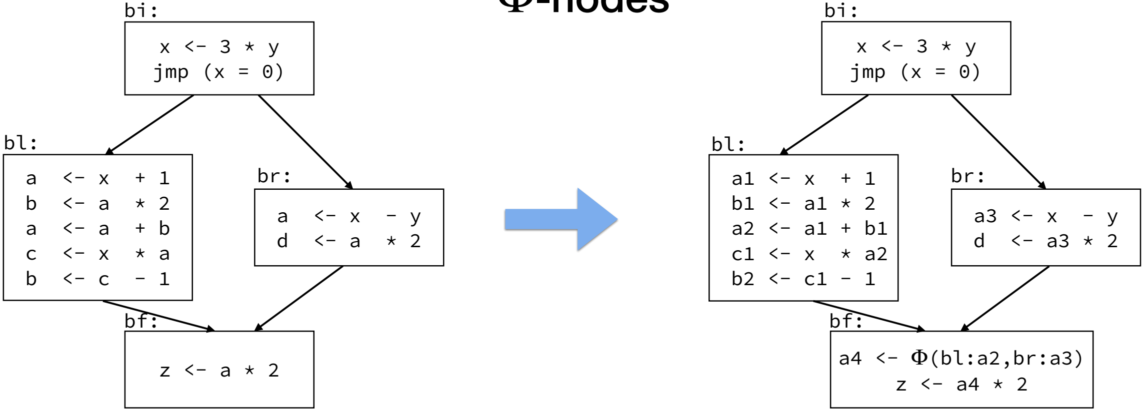
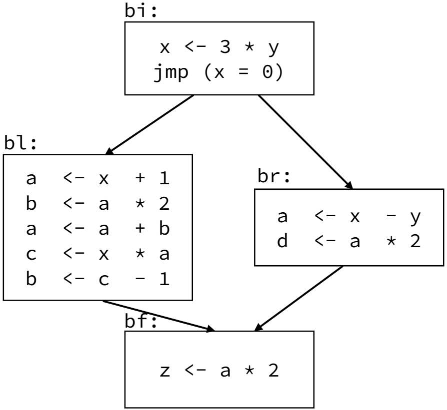
Rule 3: at merge points, introduce Φ-nodes

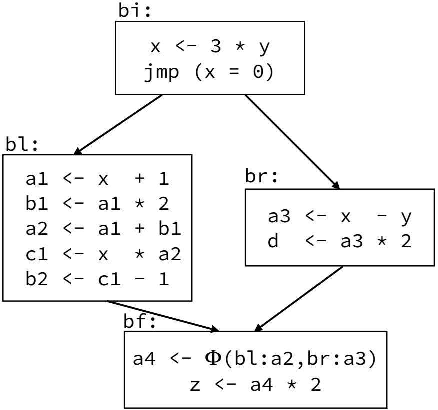
Converting merging points: Φ-nodes



Goal: to decide when to introduce Φ-nodes

Converting merging points: Φ-nodes





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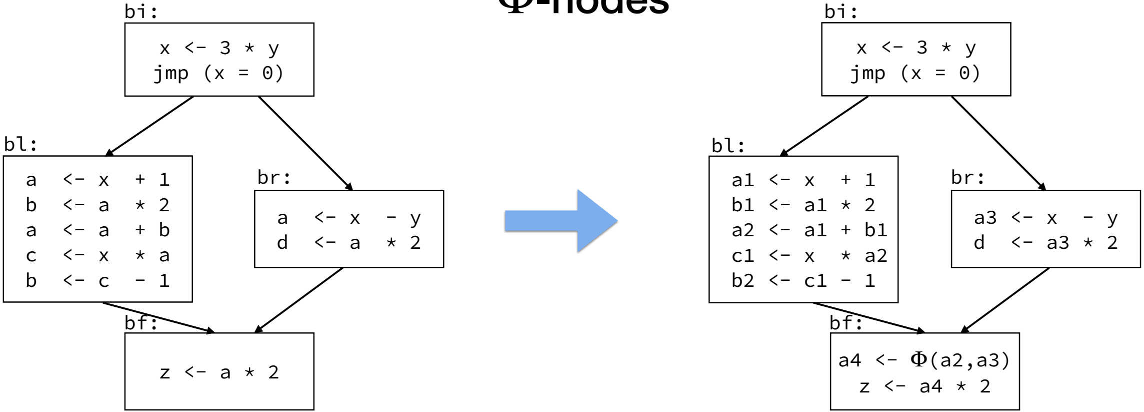
One per variable at every join point?

Converting merging points: Φ-nodes



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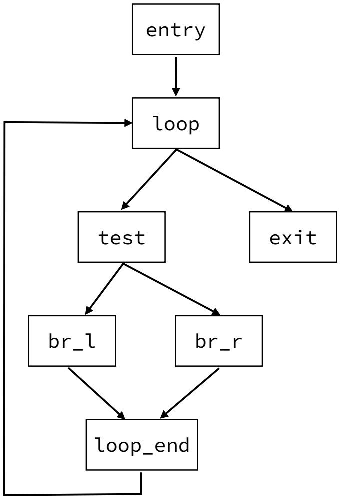
The domination relation



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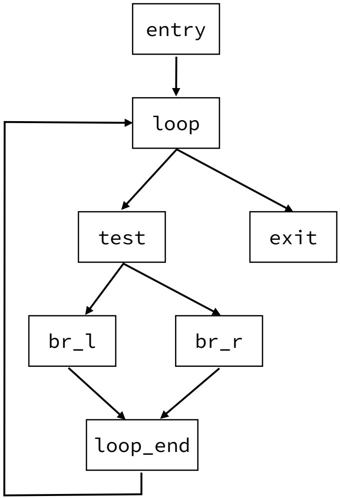

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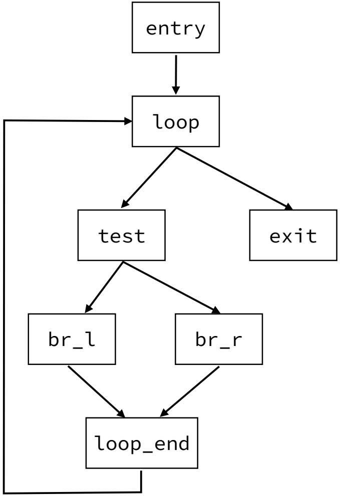
Can you annotate the nodes with their dominators?

The domination relation



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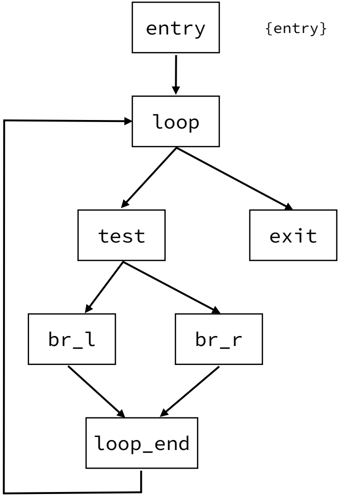

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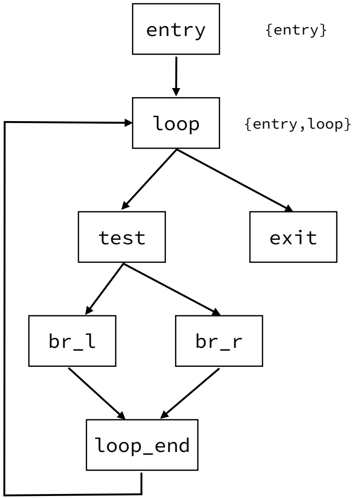

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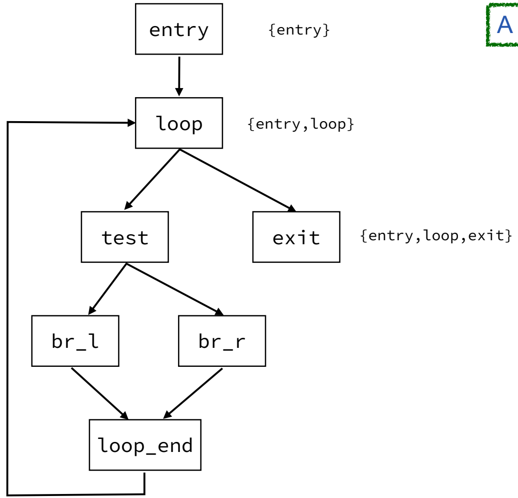

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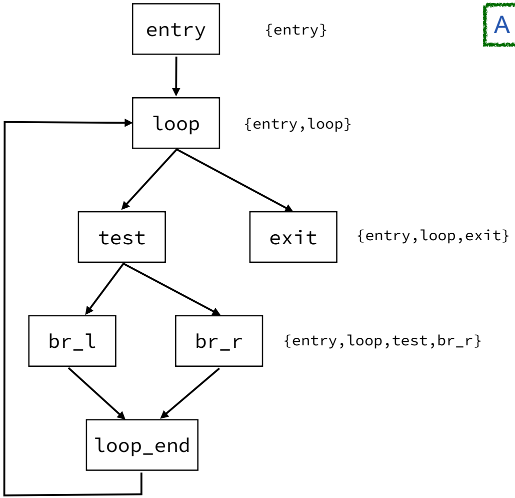

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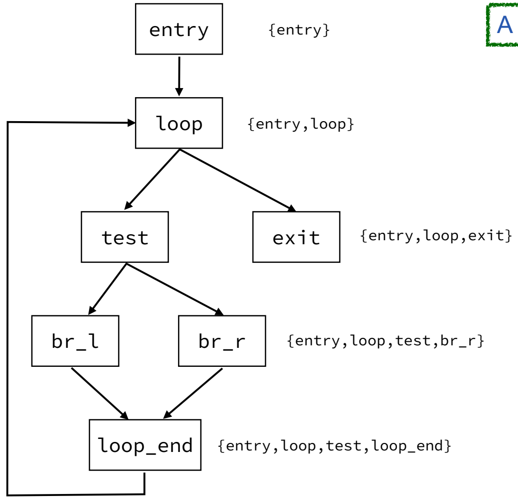

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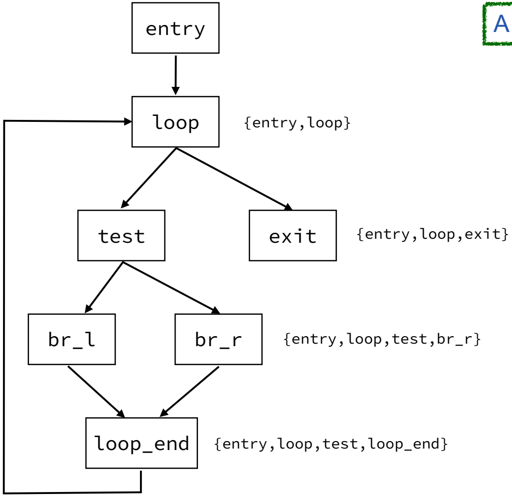

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A *strictly dominates* B if A dominates B and A is not B

The *domination tree* stores the domination relation
A is parent to B if:

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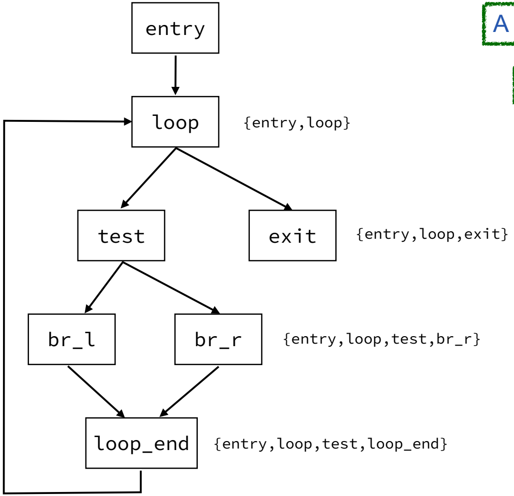

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Can you build the domination tree of the CFG to the left?

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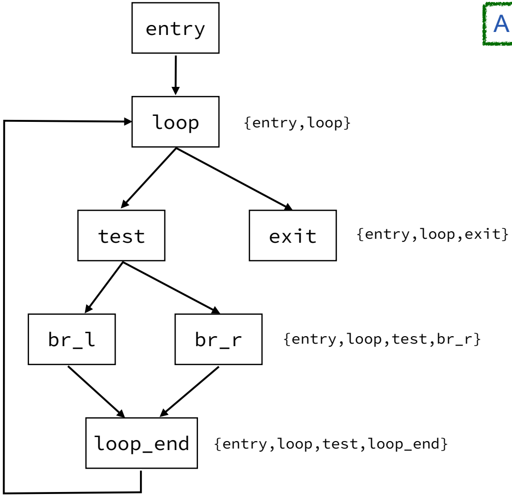

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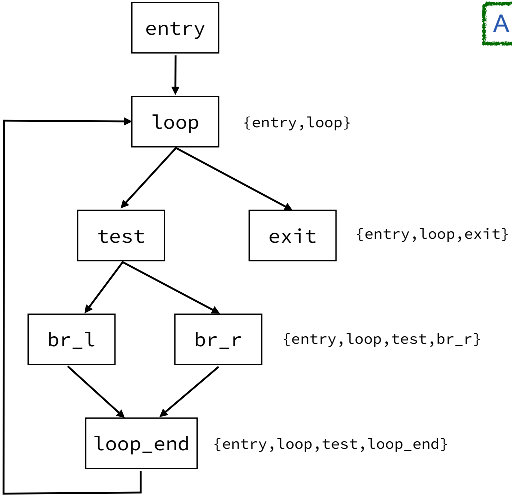

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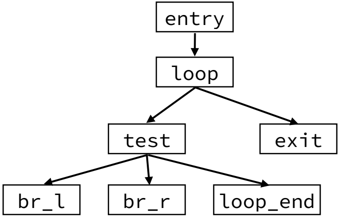
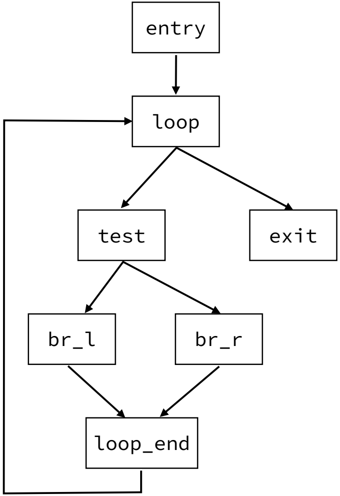


Dominance frontier



B belongs to A's *dominance frontier* if:

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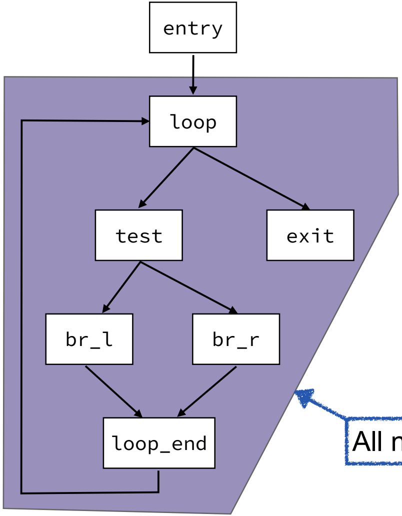

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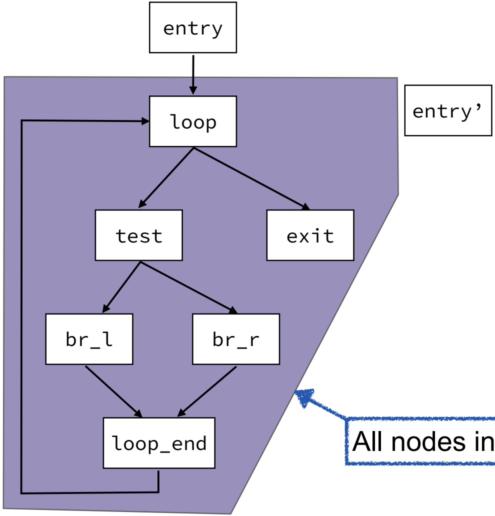

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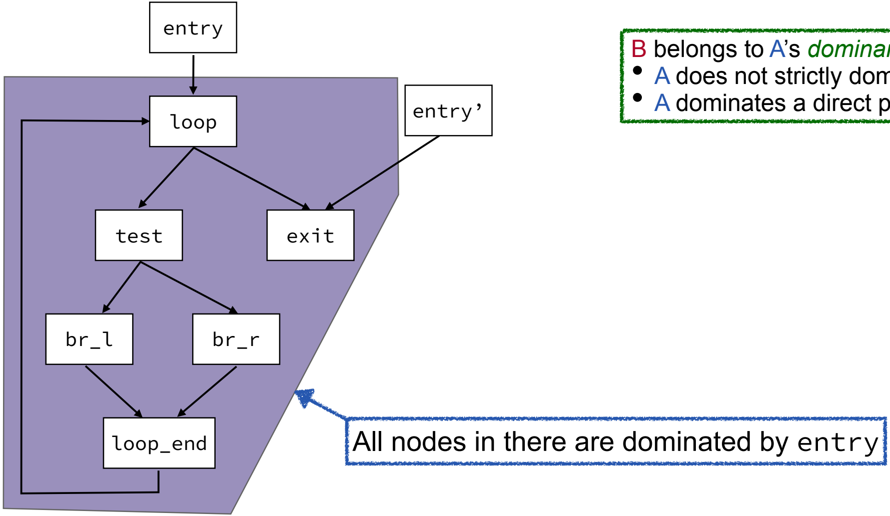

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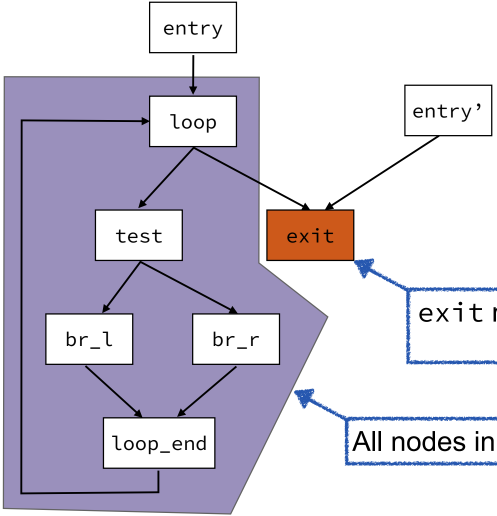

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`exit` now barely escapes the influence of `entry`: it is in its dominance frontier

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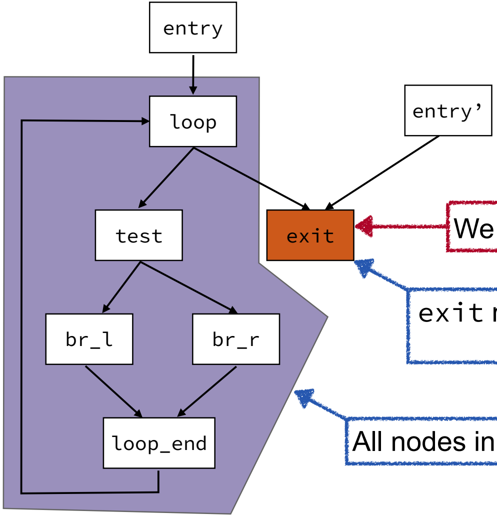
Dominance frontier



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We need a Φ-node right there!

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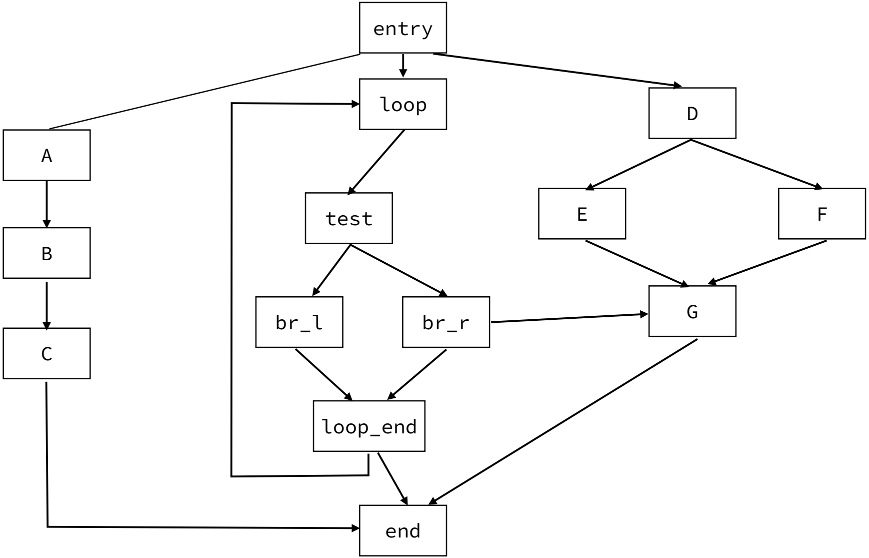

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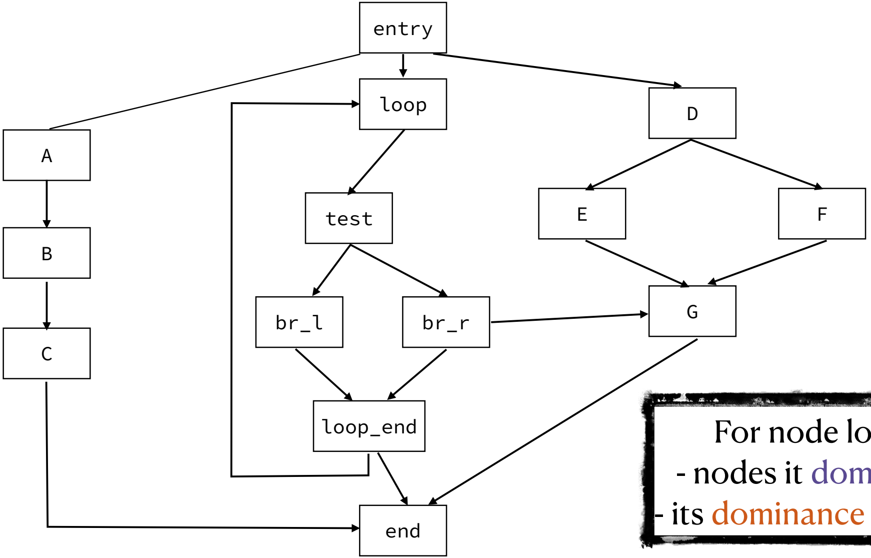
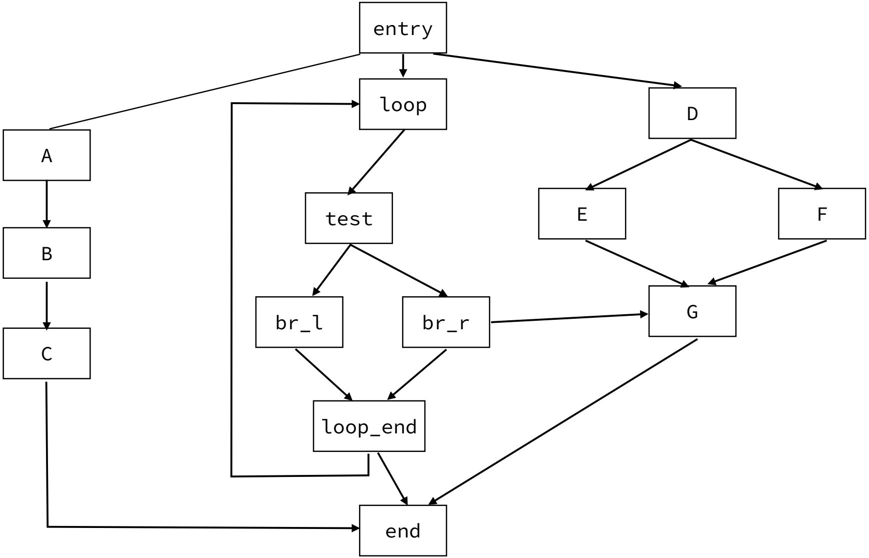


For node loop:
- nodes it dominates
- its dominance frontier

Dominance frontier



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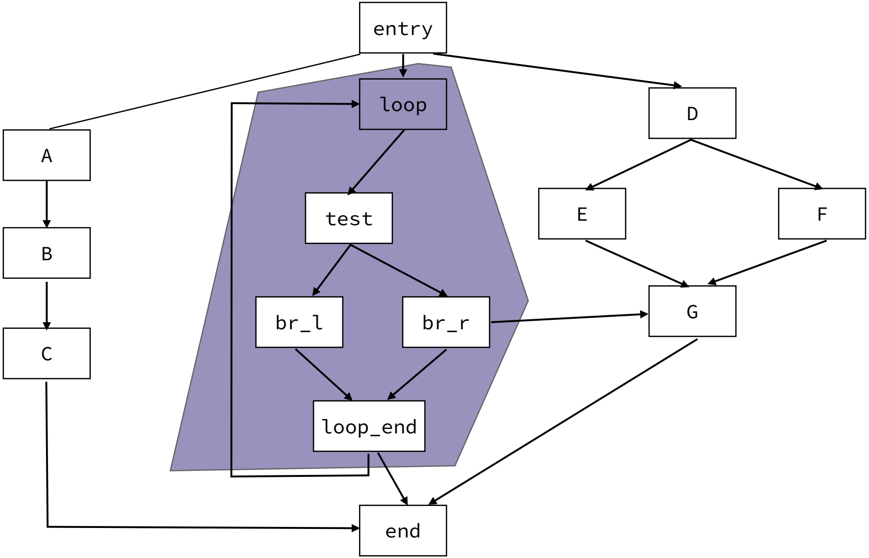

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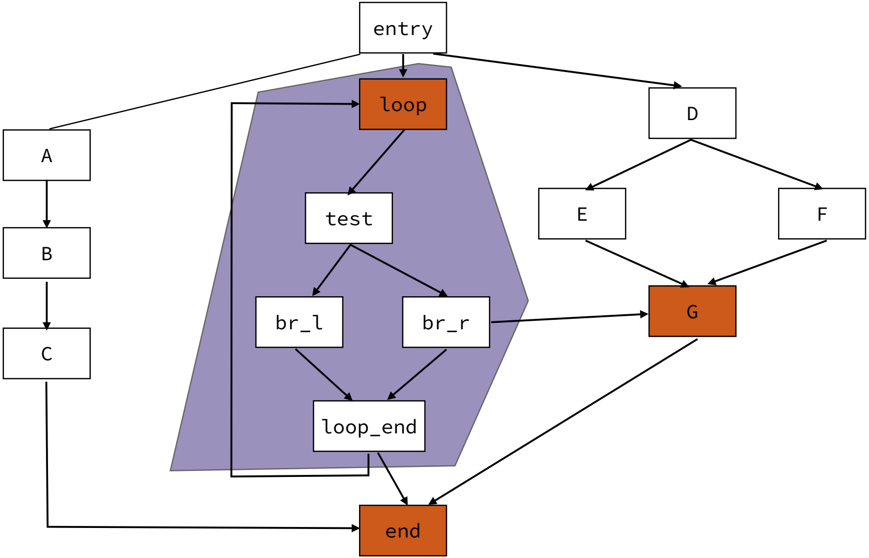

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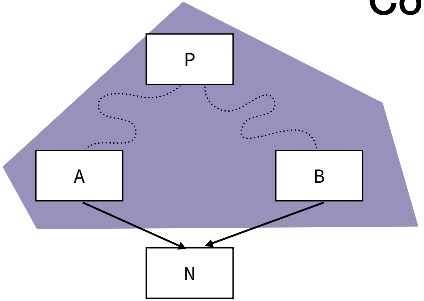

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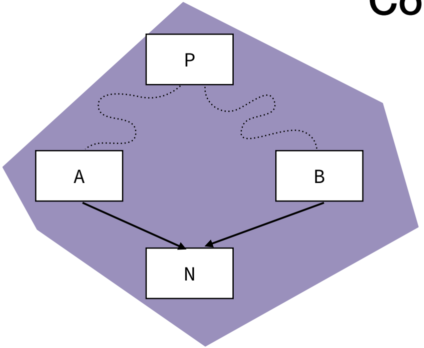

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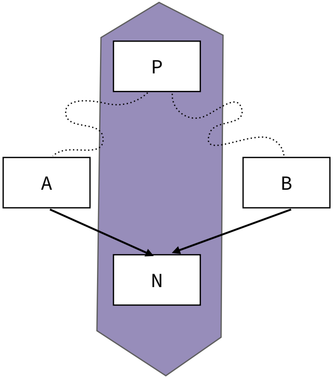

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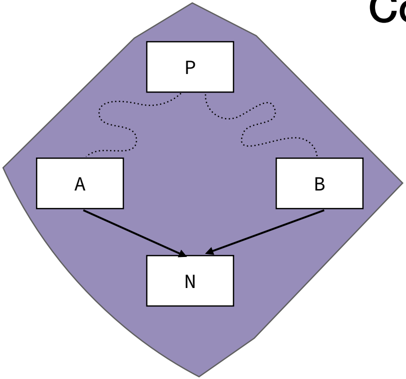

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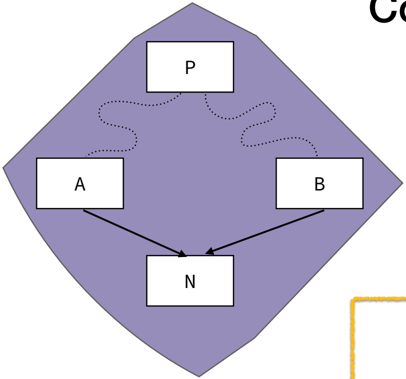

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Let D[n] be the set of nodes dominating n

$$D[entry] \triangleq \{entry\}$$

$$D[n] \triangleq \{n\} \cup \left(\bigcap_{p \in pred(n)} D[p] \right)$$

As is traditional, this system of equations can be solve by iteration[1]

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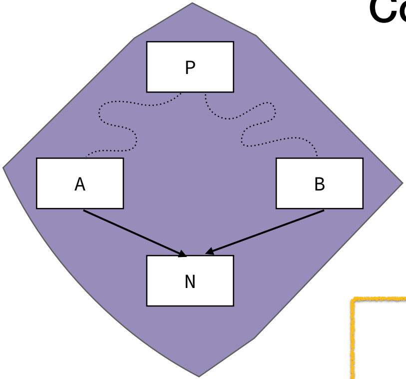

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Complexity?

[1]: For a more efficient algorithm, see Lengauer and Tarjan's 1979

"A fast algorithm for finding dominators in a flowgraph"

Computing the dominance frontier

`G` : ambient cfg
`DT`: Dominance Tree of `G`
`DF`: map from nodes to sets of nodes

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computeDF(n) ::=
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        S <- S ∪ {w}
```

The rest of the frontier is inherited from the other children

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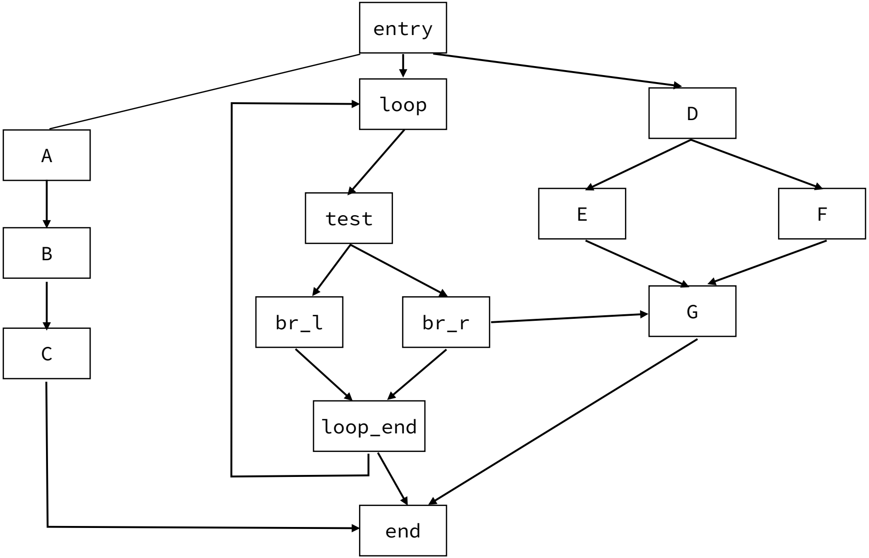
```
DF ::= computeDF(entry)
```

We kickstart the pass from the entry

Computing the dominance frontier



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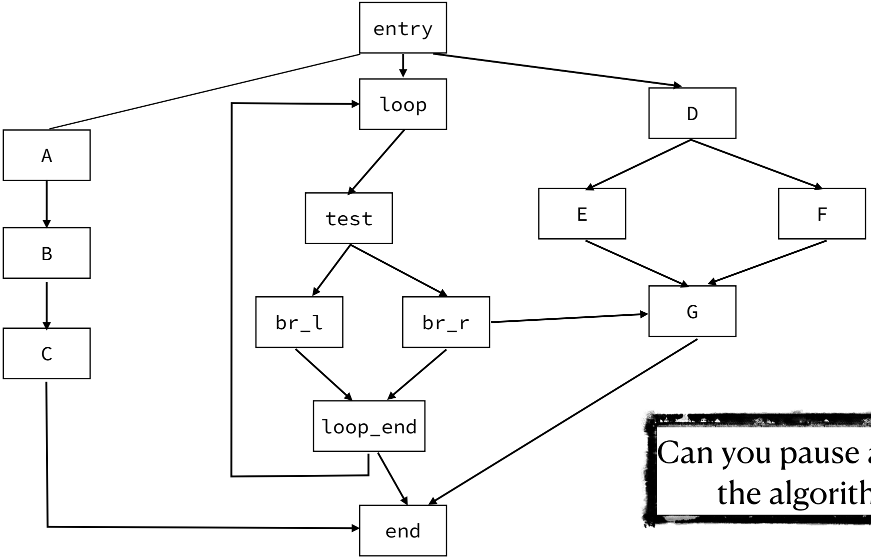
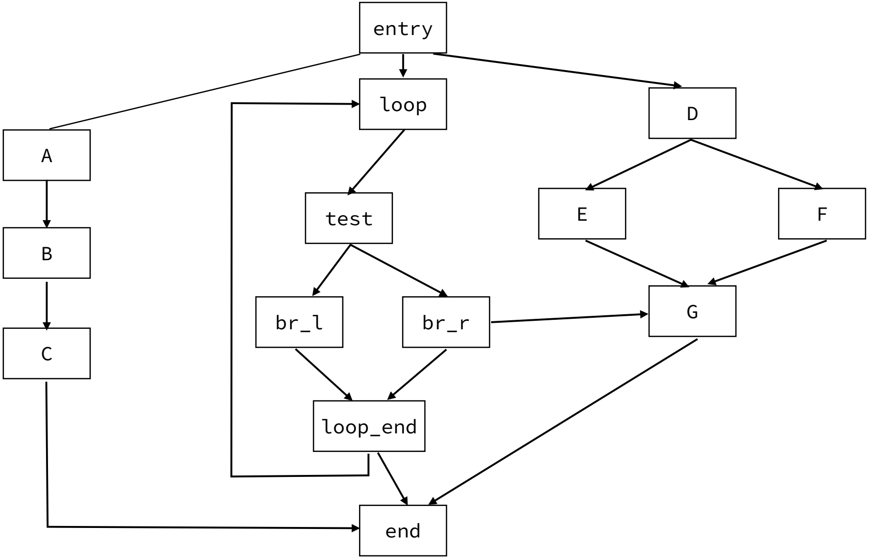


Can you pause and run the algorithm?

Computing the dominance frontier

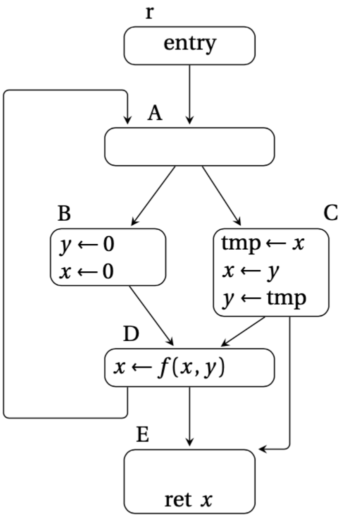


Taking stock

We want to convert a cfg to SSA-form

- The key difficulty is to figure out *where* exactly Φ-nodes are needed
- We observed *the dominance frontier* of a node seems to be the right notion
- We saw how to construct the dominance frontier,
  based on the construction of *the dominance tree*

We can now turn to the construction!



Exemple taken from the SSA book

Inserting Φ-nodes

```
Insert-phi ::=

   for x in Vars:
      for d in Defs(x):
          for b in DF(d):
              if there are no Φ-node associated to x in b:
                 add one such Φ-node
                 add b to Defs(x)
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Inserting Φ-nodes

We have not yet renamed: x can have several def-sites

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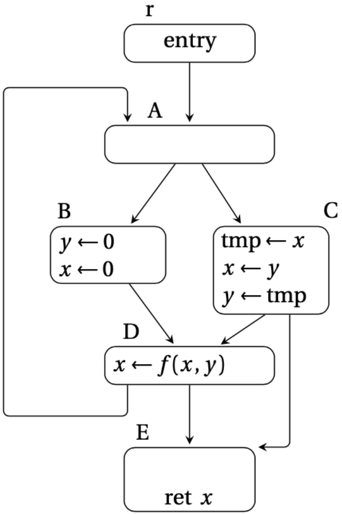
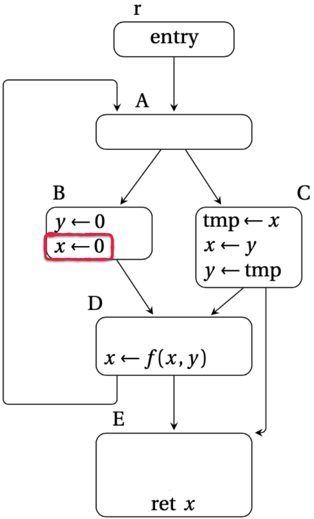
Blocks containing at least one def-site of x

A Φ-node is a new definition site!

Convince yourself it converges!





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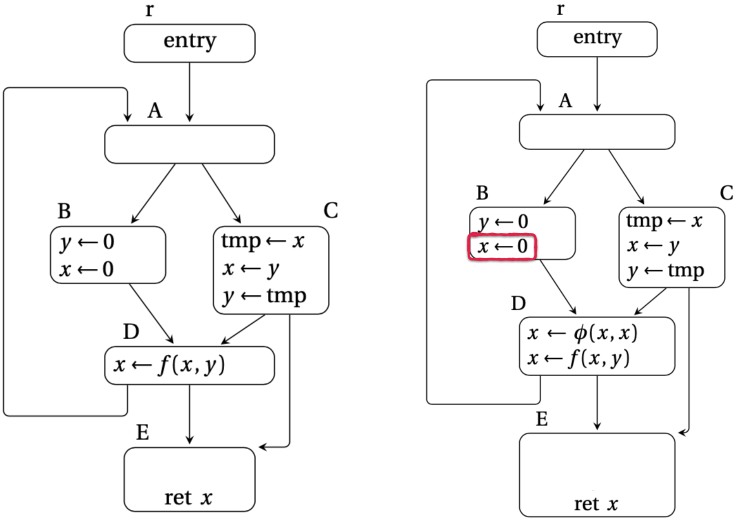

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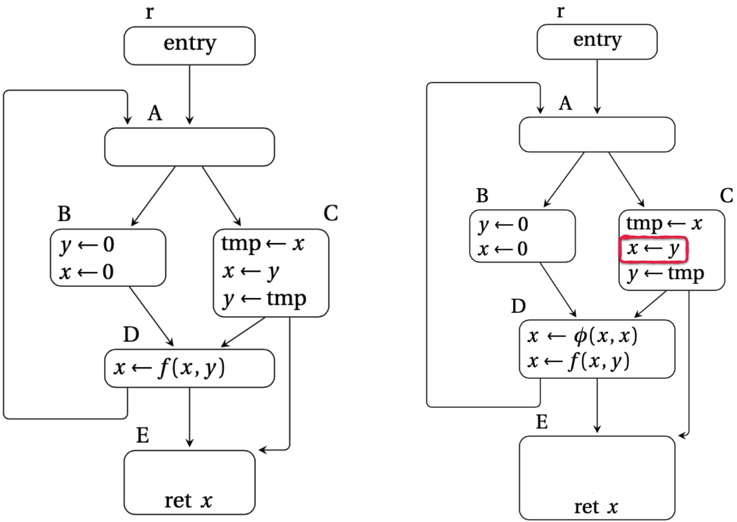

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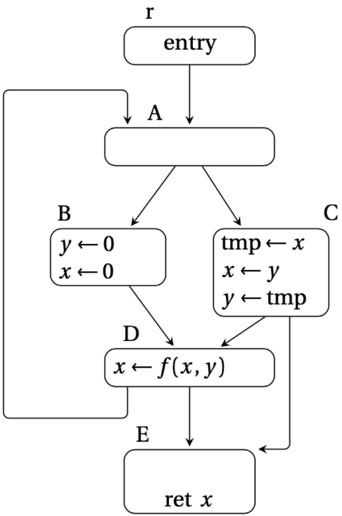

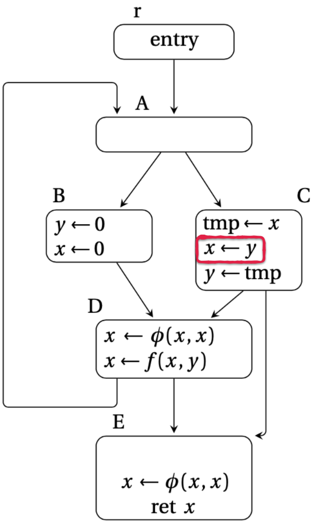

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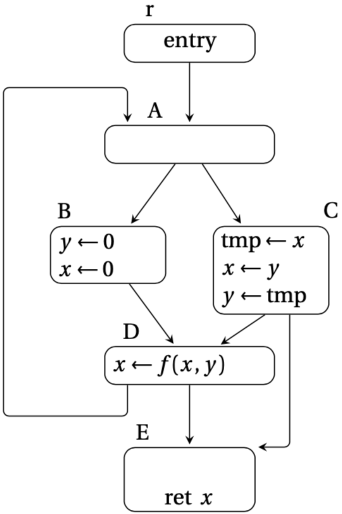

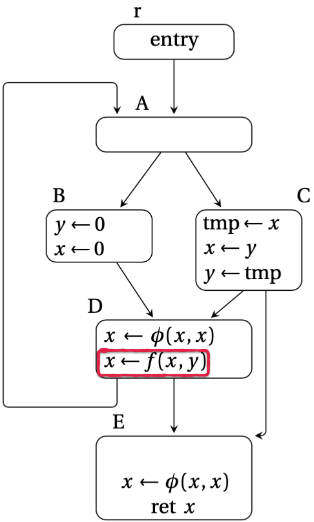

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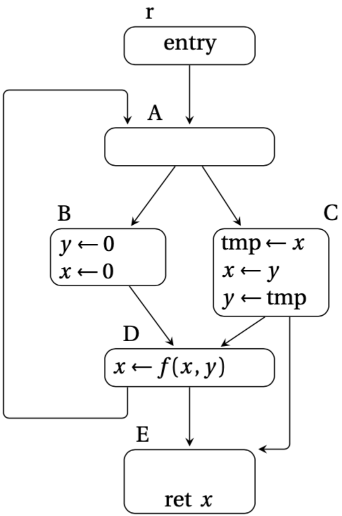

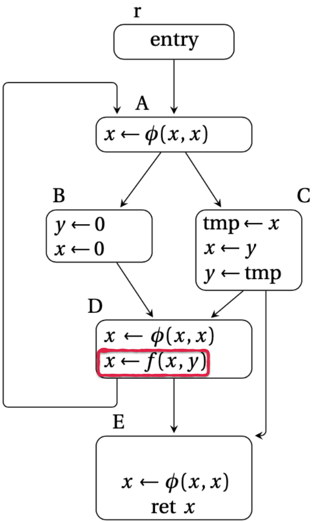

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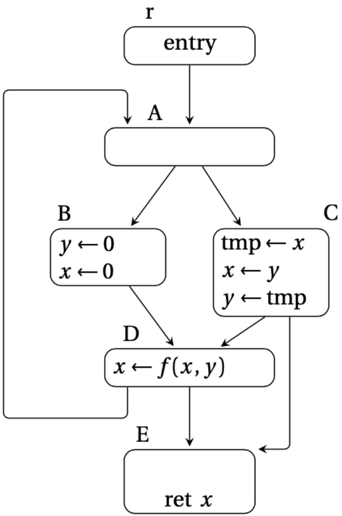

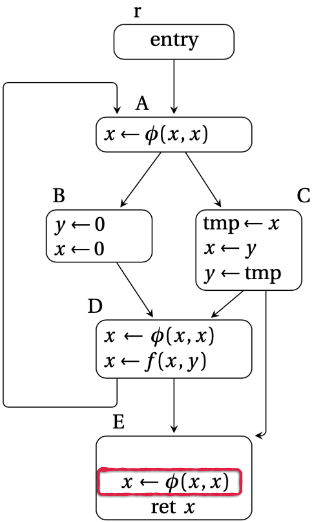

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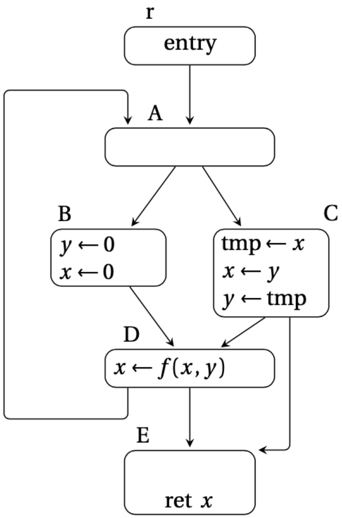

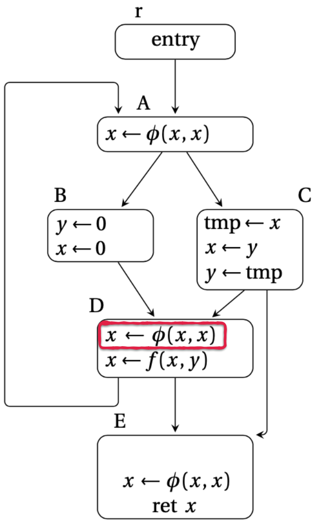

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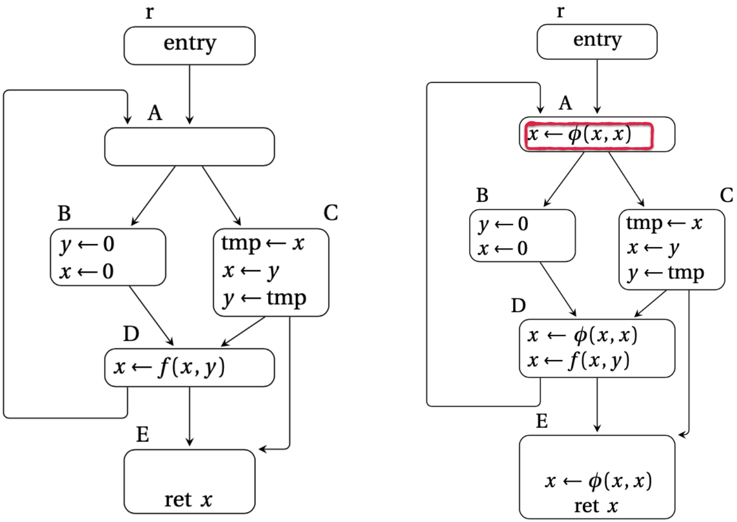

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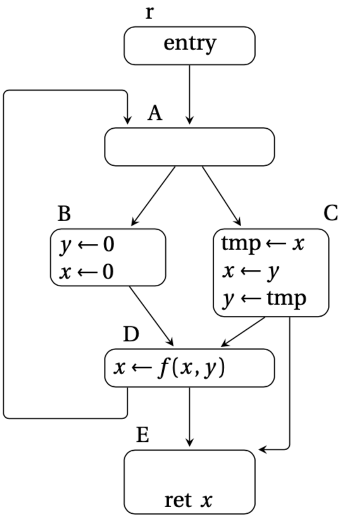

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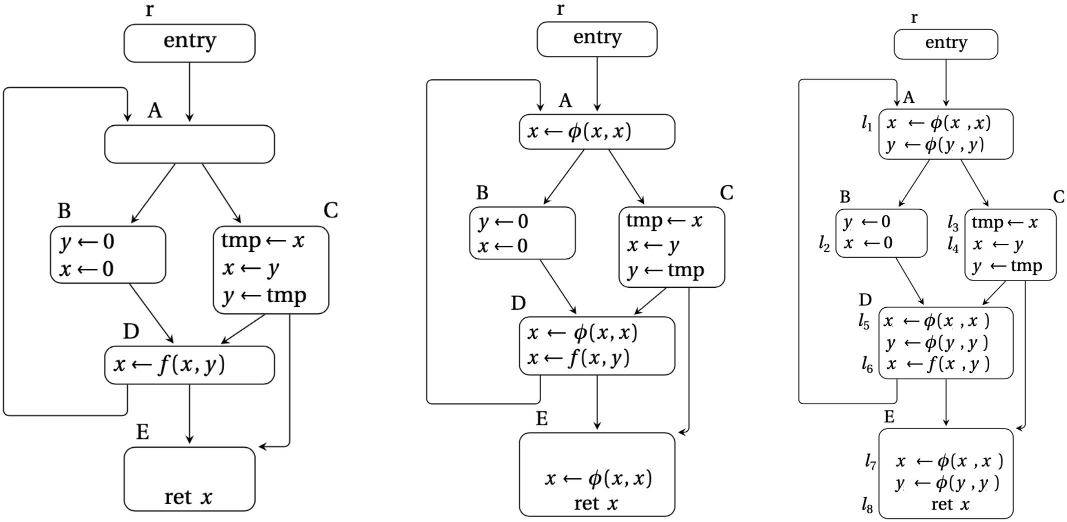

Exemple taken from the SSA book

Renaming variables

`stack[x]` : for each variable, we maintain a stack of names ("x_i")

```
rename_aux(block) ::=
```

```
rename() ::= rename_aux(entry)
```

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  for each s successor of block:
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      if x is read coming from block, replace x with stack[x]
  for each successor b of block in the DT:
    rename_aux(b)
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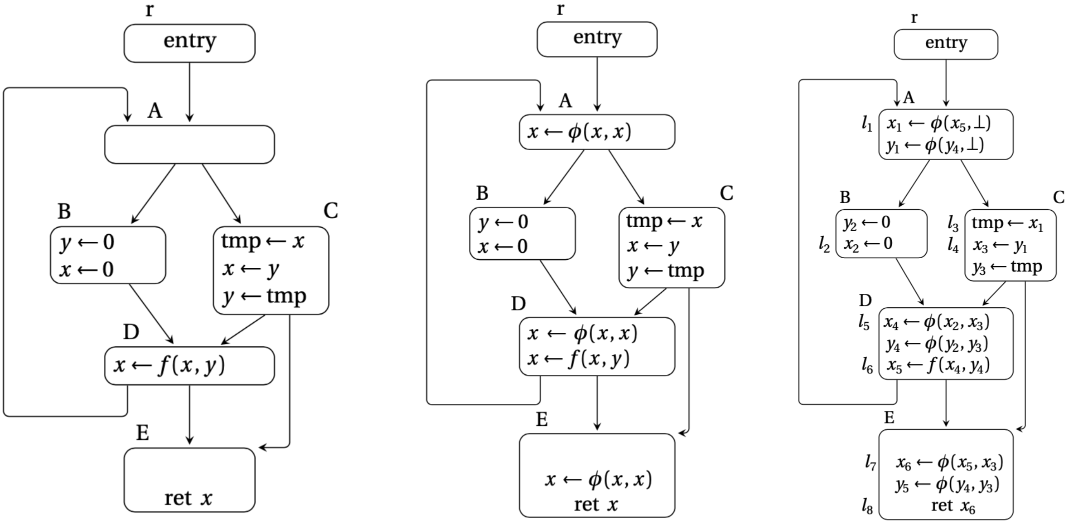
  for each successor b of block in the DT:
    rename_aux(b)
  pop from stack all variables introduced in this function call

rename() ::= rename_aux(entry)
```



Exemple taken from the SSA book

Converting out of SSA form

From SSA to machine code

Processors do not support Φ-nodes, we need to compile them away!

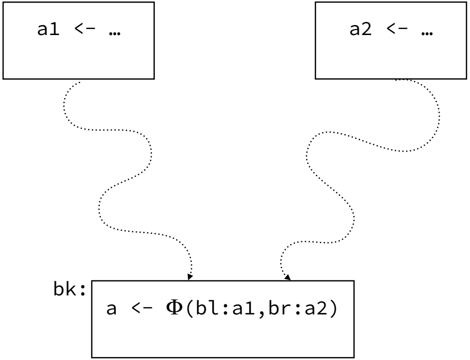
From SSA to machine code

Processors do not support Φ-nodes, we need to compile them away!

```
bk:
a <- Φ(bl:a1,br:a2)
```

From SSA to machine code

Processors do not support Φ-nodes, we need to compile them away!



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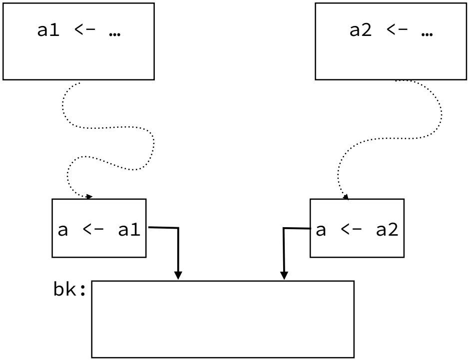
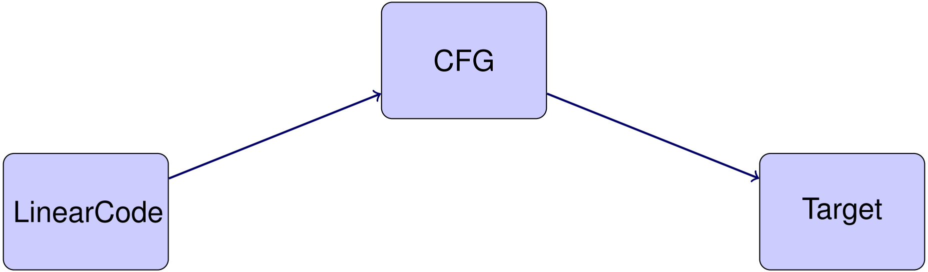


A good register allocator should then take care of eliminating needlessly introduced mov

Code Generation

In previous labs



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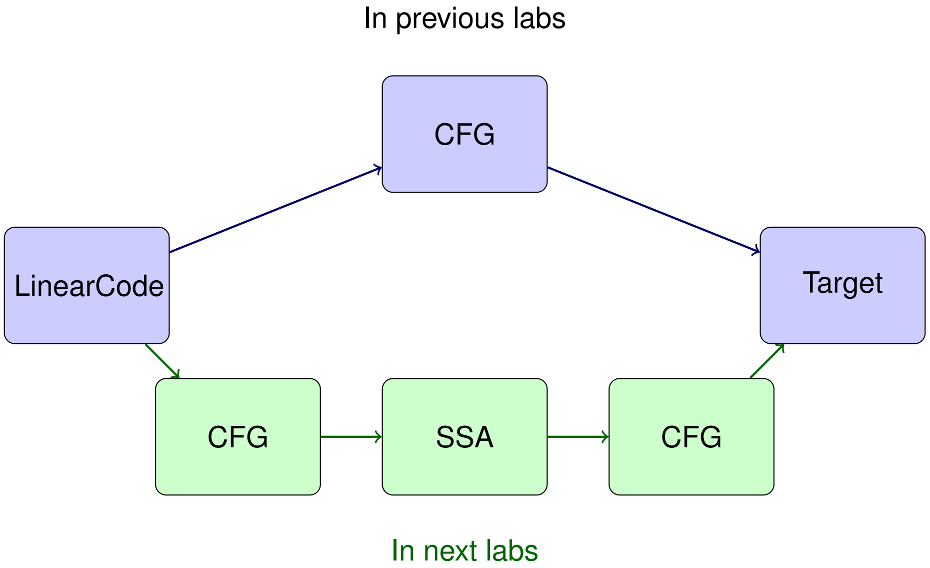

Steps

1. Implement Leader algorithm (from Linear code to CFG)
2. Implement SSA entry (dominance frontier and ϕ-insertion)
3. Implement SSA exit

To SSA and back again

```
i=1; j=1; k=0;
while (k < 100) {
  if (j < 20) {
    j=i;
    k=k+1;
  } {
    j=k;
    k=k+2;
}}
return j;
```

(Exercise taken from Fernando Pereira)

1. Draw the CFG
2. Compute the Dominance Tree and the Frontier
3. Convert to SSA
4. Convert out of SSA

To SSA and back again

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To SSA and back again

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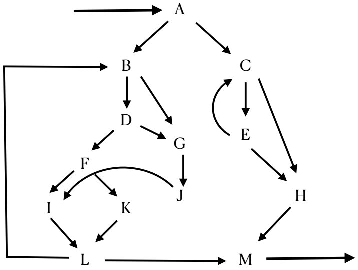
Testing dominance in constant time



We wish to test in constant time whether a given node dominates another. We assume that we have already computed the dominance tree, and allow ourself to this end a little pre-processing.

Q1. Draw the dominance tree of the graph on the right

Q2. Write an instrumented depth-first traversal labeling each node of the dominance tree with two numbers:

- N: the order in which that node was visited
- A: the maximum N among the node's descendants

Q3. Prove that these annotations can be used to test dominance in constant time.

Text adapted from an exercise designed by Fernando Pereira

Summary