EPS 2024-03-04

A. Coprime

3 seconds, 256 megabytes

Given an array of n positive integers $a_1, a_2, \ldots, a_n$ $(1 \le a_i \le 1000)$.

Find the maximum value of $i + j$ such that a_i and a_j are coprime,† or -1 if no such i, j exist.

For example consider the array $[1, 3, 5, 2, 4, 7, 7]$. The maximum value of $i + j$ that can be obtained is $5 + 7$, since $a_5 = 4$ and $a_7 = 7$ are coprime.

† Two integers p and q are coprime if the only positive integer that is a divisor of both of them is 1 (that is, their greatest common divisor is 1).

Input

The input consists of multiple test cases. The first line contains an integer t $(1 \le t \le 10)$ — the number of test cases. The description of the test cases follows.

The first line of each test case contains an integer n $(2 \le n \le 2 \cdot 10^5)$ — the length of the array.

The following line contains n space-separated positive integers a_1, a_2,..., a_n $(1 \le a_i \le 1000)$ — the elements of the array.

It is guaranteed that the sum of n over all test cases does not exceed $2 \cdot 10^5$.

Output

For each test case, output a single integer — the maximum value of $i + j$ such that i and j satisfy the condition that a_i and a_j are coprime, or output -1 in case no i, j satisfy the condition.

input

```
6
3
3 2 1
7
1 3 5 2 4 7 7
5
1 2 3 4 5
3
2 2 4
6
5 4 3 15 12 16
5
1 2 2 3 6
```

output

```
6
12
9
-1
10
7
```

For the first test case, we can choose $i = j = 3$, with sum of indices equal to 6, since 1 and 1 are coprime.

For the second test case, we can choose $i = 7$ and $j = 5$, with sum of indices equal to $7 + 5 = 12$, since 7 and 4 are coprime.

B. Matching Numbers

1 second, 256 megabytes

You are given an integer n. Pair the integers 1 to $2n$ (i.e. each integer should be in exactly one pair) so that each sum of matched pairs is consecutive and distinct.

Formally, let (a_i, b_i) be the pairs that you matched. $\{a_1, b_1, a_2, b_2, \ldots, a_n, b_n\}$ should be a permutation of $\{1, 2, \ldots, 2n\}$. Let the sorted list of $\{a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n\}$ be $s_1 < s_2 < \ldots < s_n$. We must have $s_{i+1} - s_i = 1$ for $1 \le i \le n - 1$.

Input

Each test contains multiple test cases. The first line contains the number of test cases t $(1 \le t \le 500)$. The description of the test cases follows.

For each test case, a single integer n $(1 \le n \le 10^5)$ is given.

It is guaranteed that the sum of n over all test cases doesn't exceed 10^5.

Output

For each test case, if it is impossible to make such a pairing, print "No".

Otherwise, print "Yes" followed by n lines.

At each line, print two integers that are paired.

You can output the answer in any case (upper or lower). For example, the strings "yEs", "yes", "Yes", and "YES" will be recognized as positive responses.

If there are multiple solutions, print any.

input

```
4
1
2
3
4
```

output

```
Yes
1 2
No
Yes
1 6
3 5
4 2
No
```

For the third test case, each integer from 1 to 6 appears once. The sums of matched pairs are $4 + 2 = 6$, $1 + 6 = 7$, and $3 + 5 = 8$, which are consecutive and distinct.

C. Divisible Numbers

4 s., 256 MB

You are given 4 positive integers a, b, c, d with $a < c$ and $b < d$. Find any pair of numbers x and y that satisfies the following conditions:

- $a < x \le c$, $b < y \le d$,
- $x \cdot y$ is divisible by $a \cdot b$.

Note that required x and y may not exist.

Input

The first line of the input contains a single integer t $(1 \le t \le 10)$, the number of test cases.

The descriptions of the test cases follow.

The only line of each test case contains four integers a, b, c and d ($1 \le a < c \le 10^5$, $1 \le b < d \le 10^5$).

Output

For each test case print a pair of numbers $a < x \le c$ and $b < y \le d$ such that $x \cdot y$ is divisible by $a \cdot b$. If there are multiple answers, print any of them. If there is no such pair of numbers, then print `-1 -1`.

input
```
5
1 1 2 2
3 4 5 7
8 9 15 18
12 21 14 24
36 60 48 66
```
output
```
2 2
4 6
12 12
-1 -1
-1 -1
```

D. Infinite Set

2 seconds, 256 megabytes

You are given an array a consisting of n **distinct** positive integers.

Let's consider an infinite integer set S which contains all integers x that satisfy at least one of the following conditions:

1. $x = a_i$ for some $1 \le i \le n$.
2. $x = 2y + 1$ and y is in S.
3. $x = 4y$ and y is in S.

For example, if $a = [1, 2]$ then the 10 smallest elements in S will be $\{1, 2, 3, 4, 5, 7, 8, 9, 11, 12\}$.

Find the number of elements in S that are strictly smaller than 2^p. Since this number may be too large, print it modulo $10^9 + 7$.

Input

The first line contains two integers n and p $(1 \le n, p \le 2 \cdot 10^5)$.

The second line contains n integers $a_1, a_2, \ldots, a_n$ $(1 \le a_i \le 10^9)$.

It is guaranteed that all the numbers in a are distinct.

Output

Print a single integer, the number of elements in S that are strictly smaller than 2^p. Remember to print it modulo $10^9 + 7$.

input
```
2 4
6 1
```
output
```
9
```

input
```
4 7
20 39 5 200
```
output
```
14
```

input
```
2 200000
48763 1000000000
```
output
```
448201910
```

In the first example, the elements smaller than 2^4 are $\{1, 3, 4, 6, 7, 9, 12, 13, 15\}$.

In the second example, the elements smaller than 2^7 are $\{5, 11, 20, 23, 39, 41, 44, 47, 79, 80, 83, 89, 92, 95\}$.

E_Bonus. Counting Factorizations

4 seconds, 256 megabytes

The prime factorization of a positive integer m is the unique way to write it as $m = p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots \cdot p_k^{e_k}$, where $p_1, p_2, \ldots, p_k$ are prime numbers, $p_1 < p_2 < \ldots < p_k$ and $e_1, e_2, \ldots, e_k$ are positive integers.

For each positive integer m, $f(m)$ is defined as the multiset of all numbers in its prime factorization, that is $f(m) = \{p_1, e_1, p_2, e_2, \ldots, p_k, e_k\}$.

For example, $f(24) = \{2, 3, 3, 1\}$, $f(5) = \{1, 5\}$ and $f(1) = \{\}$.

You are given a list consisting of $2n$ integers $a_1, a_2, \ldots, a_{2n}$. Count how many positive integers m satisfy that $f(m) = \{a_1, a_2, \ldots, a_{2n}\}$. Since this value may be large, print it modulo $998\,244\,353$.

Input

The first line contains one integer n $(1 \le n \le 2022)$.

The second line contains $2n$ integers $a_1, a_2, \ldots, a_{2n}$ $(1 \le a_i \le 10^6)$ — the given list.

Output

Print one integer, the number of positive integers m satisfying $f(m) = \{a_1, a_2, \ldots, a_{2n}\}$ modulo $998\,244\,353$.

input
```
2
1 3 2 3
```
output
```
2
```

input
```
2
2 2 3 5
```
output
```
5
```

input
```
1
1 4
```
output
```
0
```

In the first sample, the two values of m such that $f(m) = \{1, 2, 3, 3\}$ are $m = 24$ and $m = 54$. Their prime factorizations are $24 = 2^3 \cdot 3^1$ and $54 = 2^1 \cdot 3^3$.

In the second sample, the five values of m such that $f(m) = \{2, 2, 3, 5\}$ are $200, 225, 288, 500$ and 972.

In the third sample, there is no value of m such that $f(m) = \{1, 4\}$. Neither 1^4 nor 4^1 are prime factorizations because 1 and 4 are **not** primes.

F_Bonus. Dexterina's Lab

1 second, 256 megabytes

Dexterina and Womandark have been arch-rivals since they've known each other. Since both are super-intelligent teenage girls, they've always been trying to solve their disputes in a peaceful and nonviolent way. After god knows how many different challenges they've given to one another, their score is equal and they're both desperately trying to best the other in various games of wits. This time, Dexterina challenged Womandark to a game of Nim.

Nim is a two-player game in which players take turns removing objects from distinct heaps. On each turn, a player must remove at least one object, and may remove any number of objects from a single heap. The player who can't make a turn loses. By their agreement, the sizes of piles are selected randomly from the range $[0, x]$. Each pile's size is taken independently from the same probability distribution that is known before the start of the game.

Womandark is coming up with a brand new and evil idea on how to thwart Dexterina's plans, so she hasn't got much spare time. She, however, offered you some tips on looking fabulous in exchange for helping her win in Nim. Your task is to tell her what is the probability that the first player to play wins, given the rules as above.

Input

The first line of the input contains two integers n $(1 \le n \le 10^9)$ and x $(1 \le x \le 100)$ — the number of heaps and the maximum number of objects in a heap, respectively. The second line contains $x + 1$ real numbers, given with up to 6 decimal places each: $P(0), P(1), \dots, P(X)$. Here, $P(i)$ is the probability of a heap having exactly i objects in start of a game. It's guaranteed that the sum of all $P(i)$ is equal to 1.

Output

Output a single real number, the probability that the first player wins. The answer will be judged as correct if it differs from the correct answer by at most 10^{-6}.

input
2 2 0.500000 0.250000 0.250000
output
0.62500000